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Infrared modified analysis for the τ lepton width

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Abstract

We argue that accounting for higher order corrections in QCD by integrating a running coupling constant through an infrared region can be most easily done with making use of a scheme without the Landau pole. Within this approach the entire ambiguity of the answer is identical to that of the choice of renormalization scheme. The uncertainties for the τ lepton width resulting from such a technique are discussed.

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This talk is based essentially on our recent paper [1] and gives a brief description of our suggestion to treat those higher order corrections of perturbation theory that stem from the integration of a running coupling constant over an infrared region. Formulated in a slightly different context the problem of accounting for such corrections is known as a problem of infrared renormalons [2] and of proper determination of their contribution to physical observables. We propose the way of definition of these ambiguous contributions based on a continuation from perturbation theory region by using the freedom of the renormalization scheme choice. We parametrize this freedom with a special definition of the β function that is considered to be valid in the whole region of the copling constant (even in the regime of strong coupling). Then we introduce a particular function that gives no singularity for the corresponding running coupling constant.

Our main attention is concentrated on the analysis of the τ lepton decay width that is represented by integrals over the infrared region. Several ways to define them using the freedom of choice of the RG scheme are given [1]. The main conclusion we draw is that the results of integration can easily be made well defined without any explicit nonperturbative contributions. These results are ambiguous to the same degree as any ordinary PT series, numerically it can be important because in the infrared region the coupling constant becomes large in most of "natural" RG schemes. However it can be made small as well by some particular choice of extrapolation to low momenta.

The problem of the τ lepton width has been widely discussed in the literature (as some recent references see, e.g. [3]) so we limit ourselves to qualitatively different versions of changing RG schemes only. We propose a set of schemes that regularize the infrared behavior of the coupling constant and allow one to use any reference scheme for high energy domain. All these schemes are legal and perturbatively equivalent at high energies. Nevertheless numerical uncertainties that come from low energy region are quite essential.

We introduce our main object – a κ scheme that is determined by the following

β function [1]

$$\beta_\kappa(a) = \frac{\beta(a)}{1 - \kappa a^n \beta(a)} \tag{1}$$

where $\beta(a)$ is a β function in a reference scheme, $\overline{\rm MS}$ for instance. The form (1) is chosen because of practical convenience only – it requires no more technical work for obtaining practical results than a corresponding reference scheme and eliminates the Landau pole in infrared region.

The β function given by eq. (1) is bounded at large a and the RG equation has a solution for $a(z)$ that is defined on the whole positive semiaxis and is free of the Landau pole. The absence of singularities (Landau ghost) allows one to use the evolution of the coupling constant till the very origin. The solution to the RG equation for the invariant charge $a(z)$ in the κ scheme is simple because it is closely related to the $\overline{\rm MS}$ running coupling constant

$$ln(s/\Lambda^2) = \Phi(a) - \kappa \frac{a^{n+1}}{n+1},$$

$$\Phi(a) = \frac{1}{a} - c\ln(\frac{1}{a} + c) + \int_0^a \left(\frac{1}{\beta(\xi)} - \frac{1}{\beta_{(2)}(\xi)} \right) d\xi \tag{2}$$

with $\beta_{(2)}(a) = -a^2(1 + ca)$ and with the standard definition of the parameter Λ to be $\Lambda_{\overline{\rm MS}}$. Two charges in κ and $\overline{\rm MS}$ are connected through

$$a_\kappa = a - \frac{\kappa}{n+1} a^{n+3} + o(a^{n+3}).$$

The exact connection can be found from eq. (2). So, for $n > 1$ they coincide almost for all observables because in practice there are no calculations beyond the third order.

We now consider the uncertainties for predictions of the τ lepton width or for the parameter $\alpha_s(m_\tau^2)$ extracted from experimental data on this width. The expression for the τ lepton width has the form [4]

$$R_\tau = \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} 2(1 - \frac{s}{m_\tau^2})^2 (1 + 2\frac{s}{m_\tau^2}) R(s) \tag{3}$$

with $R(s)$ given by $R(s) = 3(1 + \frac{\alpha_s(s)}{\pi} + \ldots)$. Using the experimental value $R_\tau^{exp} = 3.645 \pm 0.024$ we find after the standard analysis in $\overline{\rm MS}$ scheme $\alpha_s(m_\tau^2) = 0.353 \pm 0.008$.

Now we give results in the κ scheme that is more general and allows the integration over infrared region. This is a generalization of the fixed point approach. Note that the analytic continuation from Euclidean region [5, 6] is a particular case of such kind an analysis. The spectral density in κ scheme (with $n = 0$) up to third order is

$$\rho(s) = a_\kappa + k_1 a_\kappa^2 + (k_2 + \kappa) a_\kappa^3, \quad a_\kappa = a - \kappa a^3, \quad a = a_\kappa + \kappa a_\kappa^3,$$

$k_1 = 0.7288$, $k_2 = -2.0314$ [7, 8]. Here the normalization of the coupling constant is chosen to be $a = \beta_0 \alpha_s$ and only the proper hadronic part of the entire spectral density is introduced (for more details see [1]). The expansion of β function reads

$$\beta(a_\kappa) = -a_\kappa^2 (1 + c a_\kappa + (c_1 - \kappa) a_\kappa^2 + \ldots)$$

$c = 64/81$, $c_1 = 3863/4374$ [9]. Thus, introducing the κ scheme is equivalent at high energies to a change of c_1. At low energies however two charges are different.

Integration for moments can be easily rewritten in terms of the charge itself

$$r_N = \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} (\frac{s}{m_\tau^2})^N \rho(a) = \int_\infty^{a_\kappa} exp(N+1)(\Phi(\xi) - \kappa\xi - \Phi(a_\kappa) + \kappa a_\kappa)(\frac{1}{\beta(\xi)} - \kappa)\rho(\xi) d\xi.$$

Introducing the variable $\zeta = 1/\xi$ we get the practical version

$$r_N = \int_0^{a_\kappa^{-1}} exp(N+1)(\Phi(\zeta^{-1}) - \kappa\zeta^{-1} - \Phi(a_\kappa) + \kappa a_\kappa) \left(\frac{1}{\zeta^2 \beta(\zeta^{-1})} + \kappa\zeta^2 \right) \rho(\zeta^{-1}) d\zeta.$$

Results depend on κ. This is the ordinary RG dependence that is strong enough because the accuracy is different at large momenta where we keep only the expansion and at small momenta where the exact formulae have to be used to make integrals finite. The obtained results are given in Table 1. Here a_κ is found from integration, α_s^{naive} is found from the naive (to third order) relation between the schemes

$$\alpha_s^{naive} = \frac{4\pi}{9}(a_\kappa + \kappa a_\kappa^3),$$

while α_s^{exact} is found from exact formulae for RG scheme relations (2). For finding the parameter a_κ from the integral it is useful to know the derivative of the integral with respect to the boundary value $a_\kappa(m_\tau^2) \equiv a_0$

$$\frac{dr}{da_0} = -\frac{1}{\beta_\kappa(a_0)} 2(r_0 - r_2 + r_3/2).$$

One can find the derivative RG equation for the coupling constant a_κ describing its dependence on the parameter κ

$$\frac{da_\kappa}{d\kappa} = a_\kappa \beta_\kappa(a_\kappa). \tag{4}$$

This is a particular case of RG equations

$$\frac{da}{dc_n} = -\beta(a) \int_0^a \frac{x^{n+3} dx}{\beta^2(x)}, \quad n \geq 1,$$

that describe the dependence of the running coupling constant on coefficients of the β function. Note that the dependence on c is fixed by the choice of the parameter Λ to be $\Lambda_{\overline{\mathrm{MS}}}$. The extraction of a_κ is done under the assumption of RG invariance of the expression for the τ lepton width so the only reliable data for a_κ can be taken from that part of Table 1 where equation (4) is satisfied. This equation can be easily solved analytically (it is a linear equation if κ is considered as a dependent function and a_κ as an independent variable), we have preferred however to solve it numerically in the vicinity of the value ($\kappa = 2.1, a_\kappa = 0.2003$). The solution does not match well the pattern of the dependence of the extracted a_κ presented in Table 1 that means that higher order terms of perturbative expansion for the width are essential.

Note that contrary to possible impression the prediction in fixed point scheme, or in K scheme (see [1]), is also nonstable. Indeed, it is easy to introduce a set of schemes parameterized with the fixed point value of the invariant charge – the extracted $\alpha_s(m_\tau^2)$ will depend on the scheme within the set. A β function for such a set could have the form

$$\beta_f(a) = \beta(a)(1 + \kappa\beta(a)) \tag{5}$$

that introduces a dependence on an external scheme parameter κ. All schemes of the type (5) have a fixed point with different value of the coupling constant depending on the parameter κ.

In fact, the dependence of the extracted values of the coupling constant for the τ lepton width on schemes is rather large because the energy scale (m_τ^2) is quite low. So, it might be reasonable to fix the scheme in an arbitrary, and somehow simplest, way and then to parameterize the low momenta region in the integral sense only without detailed description of the behavior of the running coupling from point to point. This can be done in terms of distribution. Adding the localized distribution like δ function and its derivatives one can make the integral (3) well defined [10, 11]. These localized contributions look as nonperturbative terms. Other phenomenological applications, e.g. [12], have been also considered.

To conclude, the integration over an infrared region involves the strong coupling dynamics and is arbitrary to a large extent. In case of the τ lepton width the uncertainties for extracted numerical value of the coupling constant are quite large.

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κ	a_κ	α_s^{naive}	α_s^{exact}
1.5	0.2425	0.368(16)	0.380(18)
1.6	0.2209	0.333(09)	0.342(10)
1.7	0.2118	0.318(08)	0.326(09)
1.8	0.2068	0.311(07)	0.319(08)
1.9	0.2037	0.307(07)	0.315(08)
2.0	0.2016	0.304(07)	0.313(08)
2.1	0.2003	0.303	0.312
2.2	0.1993	0.303	0.311
2.3	0.1986	0.303	0.312
2.4	0.1982	0.303	0.312
2.5	0.1979	0.303	0.313
2.6	0.1977	0.304	0.315
2.7	0.1975	0.305	0.316
2.8	0.1975	0.306	0.318
2.9	0.1975	0.307	0.319
3.0	0.1975	0.308	0.321
3.2	0.1976	0.310	0.325
3.4	0.1978	0.313	0.329
3.6	0.1980	0.315	0.334

Table 1: κ scheme results